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sources will not be proposed as problems for solution in the MONTHLY. In so far as possible, however, the editors will be glad to assist members of the Association with their difficulties in the solution of such problems.]

2966. Proposed by OTTO DUNKEL, Washington University.

When a curve produces a caustic by the reflection of the rays proceeding from a fixed point, δ and δ' the lengths of the incident and reflected rays, R the radius of curvature of the curve, and ω the angle of incidence, satisfy the equation (see *1920*, 225)

$$\frac{1}{\delta} + \frac{1}{\delta'} = \frac{2}{R \cos \omega}.$$

Give a geometrical proof of this relation using the harmonic properties of the figure and obtain in this manner a simpler and different geometrical derivation from that given in Humbert's *Cours d'Analyse*, volume 1, page 77.

2967. Proposed by ELIJAH SWIFT, University of Vermont.

A plane revolves about one of two non-coplanar lines as an axis. Find the locus, *in the plane*, of the intersection of the plane and the other line.

2968. Proposed by MALCOLM FOSTER, Yale University.

A curve C is the directrix of a ruled surface, and g is any ruling. Relative to the trihedral of C at the point of intersection with g, the direction-cosines of g are α, β, γ, expressed as functions of the arc of C. Prove that the distance t along g from C to the line of striction is given by

$$t = \frac{\frac{\beta}{\rho} - \alpha'}{\Sigma \alpha'^2 + \frac{2}{\rho}(\alpha\beta' - \alpha'\beta) + \frac{2}{\tau}(\beta'\gamma - \beta\gamma') + \frac{1}{\rho^2}(1 - \gamma^2) + \frac{1}{\tau^2}(1 - \alpha^2) + \frac{2}{\rho\tau}\alpha\gamma},$$

where ρ and τ are the radii of first and second curvature of C.

2969. Proposed by J. N. VEDDER, Union College, Schenectady, N. Y.

The following integral equations arise in connection with electrical theory:

$$f(x) = c - \lambda \int_{-a}^{+a} \log \left| \frac{a - u}{x - u} \right| f(u) du \tag{1}$$

and

$$f(x, y) = c - \lambda \int_{-a}^{+a} \int_{-b}^{+b} \log \sqrt{\frac{(a - u)^2 + (b - v)^2}{(x - u)^2 + (y - v)^2}} f(u, v) du\, dv. \tag{2}$$

The value of $\int_{-a}^{+a} f(x)dx$ is required from (1) and $\int_{-a}^{+a} \int_{-b}^{+b} f(x, y)dxdy$ from (2) in a form adapted to arithmetical calculation. In each case c is a given constant.

2970. Proposed by E. T. BELL, University of Washington.

For what values of the positive rational integers a, b, c, d, e, f are $a \cos^b (2\pi/c)$, $d \sin^e (2\pi/f)$ quadratic integers?

2971. Proposed by C. E. HORNE, The University of Porto Rico, Mayaguez, P. R.

Prove that the two tangents drawn to an ellipse from any external point subtend equal angles at a focus. (This problem is found in some text-books, but the Proposer is anxious to see an analytic solution of it.)

SOLUTIONS.

2828 [1920, 186]. Proposed by T. M. BLAKSLEE, Ames, Iowa.

On page 72 of R. B. Hayward's "*The Algebra of Coplanar Vectors and Trigonometry*" occurs the sentence: "It will be a good exercise for the student to show that cos (90°/7) $= \frac{1}{2}\sqrt{x_1}$ where x_1 is the greatest root of the equation

$$x^3 - 7x^2 + 14x - 7 = 0.$$

(1) Do not merely verify but deduce the equation and find x_1. (2) Deduce the x-equation (x_1, x_2, x_3, x_4 the roots) such that its greatest root x_1 gives $\cos (90°/9) = \cos 10° = \frac{1}{2}\sqrt{x_1}$. (3) Of what angles are $\frac{1}{2}\sqrt{x_1}, \cdots \frac{1}{2}\sqrt{x_4}$ in (2) the cosines? Develop a method of writing out at once $\cos nv$ in terms of powers of $\cos v$ if these are given for $(n-1)v$ and $(n-2)v$. The same for $\sin nv$. (4) Use the results of (2) and (3) to find the number of degrees in a radian. Hence, find π from radian instead of radian from π as is usual.

SOLUTION BY J. B. REYNOLDS, Lehigh University.

From the formula

$$\cos x + \cos y = 2 \cos \tfrac{1}{2}(x+y) \cos \tfrac{1}{2}(x-y),$$

letting $x = nv$ and $y = (n-2)v$, we have[1]

$$\cos nv = 2 \cos v \cos (n-1)v - \cos (n-2)v.$$

So also $\sin nv = 2 \cos v \sin (n-1)v - \sin (n-2)v$.
Putting $n = 2, 3, \cdots 9$, we readily obtain

$$\cos 7v = 64 \cos^7 v - 112 \cos^5 v + 56 \cos^3 v - 7 \cos v,$$

and

$$\cos 9v = 256 \cos^9 v - 576 \cos^7 v + 432 \cos^5 v - 120 \cos^3 v + 9 \cos v.$$

Replacing in these $\cos v = \frac{1}{2}\sqrt{x}$, we get

$$\cos 7v = \frac{x^{1/2}}{2}(x^3 - 7x^2 + 14x - 7), \tag{1}$$

$$\cos 9v = \frac{x^{1/2}}{2}(x^4 - 9x^3 + 27x^2 - 30x + 9). \tag{2}$$

If in (1), we let $\cos 7v = 0$ or $7v = (n + \frac{1}{2})\pi$, we have $v = \pi/14 = 90°/7$ for the greatest root $4 \cos^2 v$, of

$$x^3 - 7x^2 + 14x - 7 = 0. \tag{3}$$

Again, if we let $\cos 9v = 0$ or $9v = (n + \frac{1}{2})\pi$, we have $v = (2n+1)\pi/18$ for the roots of

$$x^{1/2}(x^4 - 9x^3 + 27x^2 - 30x + 9) = 0; \tag{4}$$

and if we call them x_1, x_2, x_3, x_4, 0, for $n = 0, 1, 2, 3, 4$, then

$$x_1 = 4 \cos^2 \pi/18, \quad x_2 = 4 \cos^2 \pi/6, \quad x_3 = \cos^2 5\pi/18, \quad x_4 = 4 \cos^2 7\pi/18, \quad x_5 = 4 \cos^2 \pi/2 = 0.$$

Since $4 \cos^2 \pi/6 = 3$, we may factor (4) into

$$x^{1/2}(x-3)(x^3 - 6x^2 + 9x - 3) = 0. \tag{5}$$

We may solve (3) and (5) by Horner's method to get any of the roots to the desired degree of accuracy, since the roots are all real. In this manner, we find from (5) $x_1 = 3.87938$, $x_3 = 1.65271$, and since $x_1 + x_3 + x_4 = 6$, $x_4 = .46791$, $x_2 = 3$. Now we have $\cos 70° = \frac{1}{2}\sqrt{x_4}$ or

$$70° = \cos^{-1}(\tfrac{1}{2}\sqrt{x_4}) = 90° - \sin^{-1}(\tfrac{1}{2}\sqrt{x_4}),$$

giving

$$20° \text{ in radians} = \sin^{-1}\left(\frac{x_4^{1/2}}{2}\right) = \frac{x_4^{1/2}}{2}\left\{1 + \frac{x_4}{24} + \frac{3x_4^2}{640} + \frac{5x_4^3}{64 \cdot 112} + \cdots\right\}$$

$$= .349058 \text{ radians};$$

whence, 1 radian = 57.297° or $\pi/9 = .349058$, giving $\pi = 3.14152$.

2830 [1920, 227]. Proposed by WILLIAM HOOVER, Columbus, Ohio.

An elastic string connects a pair of opposite vertices of a square whose sides are four equal rods freely jointed, each of length $2a$. The system is suspended vertically from a vertex attached to the string and is at rest. If the string be cut, and θ is the acute angle any side makes with the vertical at any moment during the motion, determine the angular velocity of any rod.

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[1] Compare Note 3, in this MONTHLY, *1921*, 38–39.